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# **Benchmarking Railway Vibrations - Track, Vehicle, Ground and Building Effects**

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**Keywords:** *Passenger comfort, railway track dynamics, underground train, high speed rail, urban tram, critical velocity, structural vibration, railway vibration, environmental assessment, EIA, track displacements, soil parameter assessment, site investigation*

## **Abstract**

This paper reviews, synthesises and benchmarks new understandings relating to railway vibrations. Firstly, the effect of vibrations on passenger comfort is evaluated, followed by its effect on track performance. Then ground-borne vibration is discussed along with its effect on the structural response of buildings near railway lines. There is discussion of the most suitable mathematical and numerical modelling strategies for railway vibration simulation, along with mitigation strategies. Regarding ground borne vibration, structural amplification is discussed and how vibration mitigation strategies can be implemented. There is also a focus on determining how ‘critical velocity’ and ‘track critical velocity’ are evaluated – with the aim of providing clear design guidelines related to Rayleigh wave velocity. To aid this, conventional site investigation data is reviewed and related to critical velocity calculations. The aim is to provide new thinking on how to predict critical velocity from readily available conventional site investigation data.

## **Introduction**

Over the last 50 years there has been an increasing demand for more access to the railways for both passenger and freight carriage. One emerging way to satisfy this demand for increased capacity has been to create high speed passenger routes – thus freeing up capacity on the existing networks for classic rail serving urban conurbations and providing extra capacity for freight trains. Figure 1 illustrates the evolution of railway train speeds since the 1960’s.

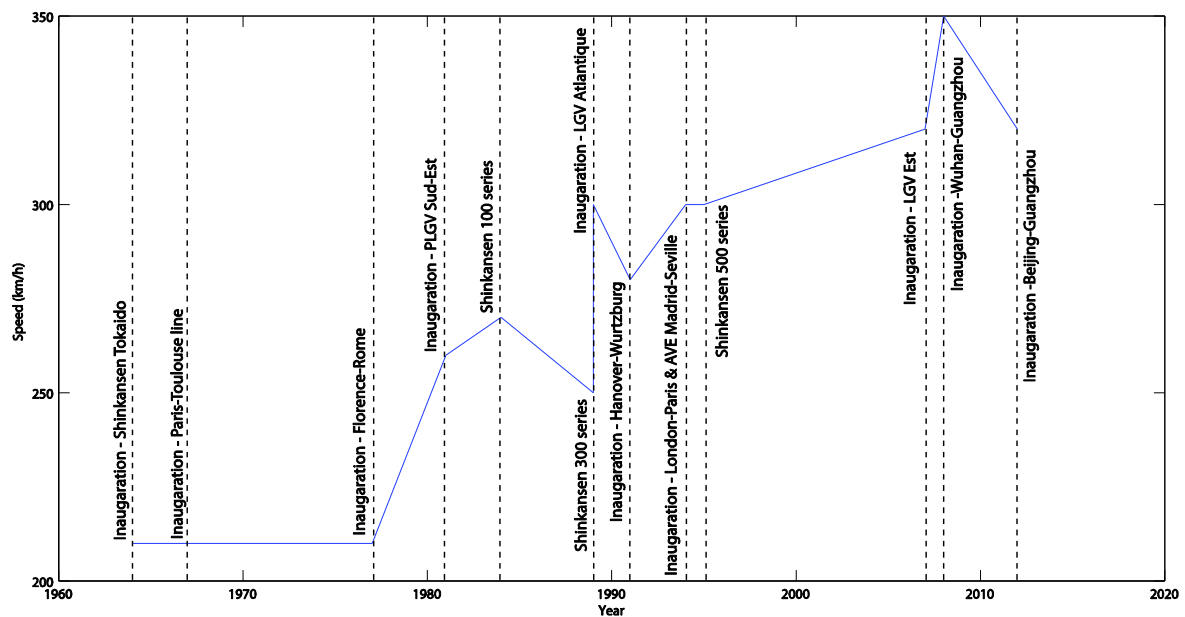


Figure 1 - Historical maximum train speed timeline (data taken from [1])

With the creation of this extra capacity, much at high speeds - railway vibration is now a growing engineering challenge due to the higher speeds and heavier loads, in close proximity to densely populated urban environments. These faster or heavier trains impart greater forces into the track and can result in elevated vibration levels within both train and track, thus effecting passenger safety, maintenance costs and passenger comfort. In addition, when these vibrations propagate outward from the track they interact with their surrounding environment, which can cause negative side effects, particularly in urbanised areas.

This paper attempts to provide a comprehensive, detailed review of the vibrations generated within the train, track, ground and nearby structures, with each component being reviewed separately. For each element, vibration generation/propagation is described, along with practical considerations, mitigation possibilities and potential modelling approaches. It aims to do so in a manner that is useful for both academics and practitioners.

Firstly, as the wheel-rail interface is the source of vibration, the mechanisms that generate vibrations are discussed, along with general wave propagation theory. Then vibration propagation within the train vehicle is reviewed with a focus on passenger comfort which is becoming increasingly important on new lines. Next the role of the track is considered with a focus on numerical modelling and common vibration mitigation procedures. The role of the ground in the transfer of vibration from track to nearby structures is also analysed with a focus on modelling and critical velocity effects. Lastly, building vibration and the generation of in-door noise is reviewed. It should be noted that this work is complementary to other state of the art reviews into railway vibration, as listed in Table 1 below.

Topic of review paper	Reference
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Railway vibration - modelling approaches	[2], [3], [4]
Track settlement	[5]
Track loading conditions	[6], [7], [8], [9]
Vehicle dynamics	[10]
Track vibration mitigation	[11]
Tunnel vibration	[12]
Building vibration	[13], [14]
Passenger comfort	[15]

Table 1 - Topics of previous review papers

## Background

### Vibration generation

Railway vibration and noise arise from the forces generated at the contact point between train wheel and the rail. These forces can be broken down into their quasi-static and dynamic components [16]. Quasi-static forces arise from the train weight and are independent of train speed. They dominate the track response and near field, at distances up to one quarter of a wavelength [17]. If the track is considered as a Euler beam resting on an elastic foundation, the quasi static deflection of a typical track is shown in Figure 2 and can be calculated analytically by

$$w(x, t) = w(x - v_0 t) = [\cos(\beta|x - v_0 t|) + \sin(\beta|x - v_0 t|)] \frac{P + m_w g}{8E_r I_r \beta^3} e^{-\beta|x - v_0 t|}$$

Where  $w(x, t)$  is the track deflection at position 'x' and time 't'. ' $E_r$ ' and ' $I_r$ ' are the Young's modulus and second moment of area of the Euler beam respectively. The train load is of constant force 'P', moving at a constant speed ' $v_0$ ', and ' $K_f$ ' is the stiffness per unit length of the Winkler foundation.  $\beta$  is defined as:

$$\beta = \sqrt[4]{\frac{K_f}{4E_r I_r}}$$

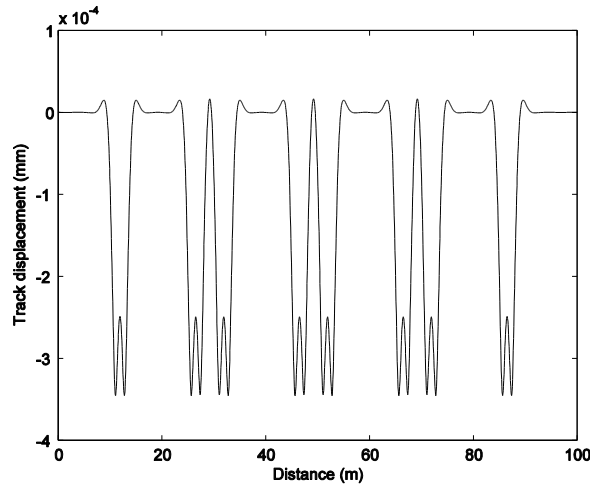


Figure 2 – Quasi-static track deflection

In contrast, dynamic excitation is speed dependent and arises from factors such as changes in stiffness due to sleeper placement, irregularities at the wheel/rail interface and the soil support conditions. Dynamic excitation from irregularities and rail joints is slowly becoming a less influential excitation mechanism due to the widespread use of continuously welded rails and improved track maintenance.

Although all railway lines generate vibration, differences between train/track features mean that the characteristics of the generated vibrations vary widely. Historically, four cases have been of particular concern:

**Underground trains** – generate vibrations with a higher frequency spectrum than over-ground tracks. Although noise is also generated, it is contained within the tunnel and therefore vibrations are of primary concern for structures located above the line.

**High speed trains** – generate elevated amplitude vibrations due to their increased speeds. Additionally, if their speed becomes comparable to the wave speed in the supporting soil then vibration levels may become magnified.

**Urban tramways** – Generate relatively low amplitude vibrations, however their close proximity to buildings can cause negative structural effects. Furthermore, increases in unsprung mass due to the more frequent deployment of low-floor vehicles has exacerbated vibration problems.

**Freight trains** – generate high amplitude, low frequency vibration (due to their low speed) that can propagate to large distances from the track.

### Wave propagation

Upon generation, the vibrations caused at the wheel/rail interface propagate through the track and into the free field in the form of waves. These waves are categorised as either body waves or surface waves. Surface waves travel along a structures (i.e. soil) surface and decay exponentially with depth. Body waves propagate primarily beneath the soil surface. Wave propagation characteristics are shown in Figure 3.

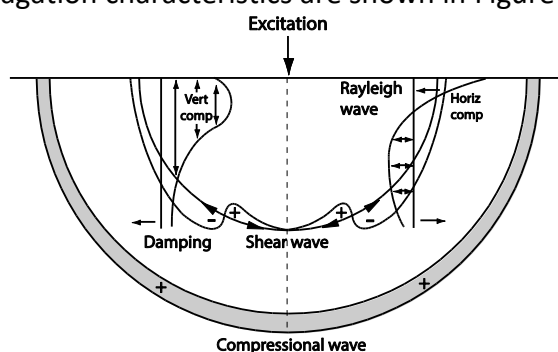


Figure 3 - Seismic wave propagation (slice view) [after 18]

Compressional waves (P-waves) propagate in a longitudinal direction and travel faster than all other types of waves. Shear waves (S-waves) propagate in a transverse direction and although they travel faster than Rayleigh waves, they always travel slower than P-waves. Rayleigh waves are the slowest type of seismic wave, the speed of which can

be approximated using numerous formulas ([19], [20], [21]). The analytical formulas to calculate P and S wave speeds are:

$$C_p = \sqrt{\frac{\lambda + 2\mu}{\rho}}$$

$$C_s = \sqrt{\frac{\mu}{\rho}}$$

Where  $\rho$  is density,  $\lambda$  is bulk modulus and  $\mu$  is the shear modulus ( $\lambda$  and  $\mu$  are also known as Lamé's parameters).

Although other types of waves are theoretically possible (e.g. Lamb waves in layers and Stoneley waves at interfaces), compressional, shear and Rayleigh are the most common and are the focus of this research. Furthermore, an emphasis is placed on the propagation of Rayleigh waves as they transmit approximately two thirds of the total excitation energy (Rayleigh waves $\approx$ 67%, S-waves $\approx$ 26%, P-waves $\approx$ 7% [18]). Therefore they are most likely to cause negative effects in both the railway track and nearby structures.

There are a wide variety of parameters used to classify soil material. Different parameters are used in different circumstances and determined in different ways. Table 2 lists some of these parameters and their context, where S.I. is Site Investigation:

Soil parameter	Soil type	Purpose or application	Obtained in traditional S.I.
Liquid limit	Clay	Classification	Yes
Plastic limit	Clay	Classification	Yes
Moisture content	Granular & clay	Classification	Yes
Density	Granular & clay	Classification + numerical analysis	Yes
Particle size distribution	Granular & clay	Classification	Yes
Shear strength	Granular & clay	Design: foundation & slope stability	Yes
Over-consolidation ratio	Granular & clay		No
Poisson's ratio	Granular & clay	Numerical analysis	No
Young's modulus	Granular & clay	Numerical analysis	No
Damping	Granular & clay	Numerical analysis	No

Table 2 – Material properties

For numerical analysis wave propagation can be efficiently described using four main material properties: Density, Poisson's ratio, Young's modulus and damping. Although more traditionally measured soil characteristics such as moisture content, particle size distribution, liquid and plastic limits, consolidation ratio, etc. affect material characteristics (and thus wave propagation), their effect is usually accounted for by changes in the values of the aforementioned parameters.

The real challenge is that the conventional site investigation does not measure Density, Poisson's ratio, Young's modulus and damping. This aspect will be explored later in the paper and a proposal made to relate conventional site investigation data to required numerical analysis input data.

### Review of numerical analysis input data

**Density** – Measured in a conventional SI - the mass divided by the unit volume of a material. Density typically increases with depth because lower soil layers tend to have experienced elevated consolidation and therefore the solid particles are more tightly packed together.

**Poisson's ratio** – When a material is compressed using a force in a single direction, Poisson's ratio defines the degree to which the material expands in the other two directions. This is the ratio of expansion to the contraction caused by the compression.

Sudden increases of Poisson's ratio within a soil are often due to the presence of the water table. This is particularly true for clays which when fully saturated become incompressible (i.e.  $\nu \approx 0.5$ ). In this case the P-wave speed increases dramatically because the wave speed becomes more representative of the water rather than the soil. On the other hand the S-wave velocity remains unchanged because water has no shear strength and thus the wave speed remains representative of the soil. Changes in wave speed with respect to Poisson's ratio are shown in Figure 4. It can be noticed that Poisson's ratio also has an effect on Rayleigh wave speed. This effect is minor because the Rayleigh wave speed can never exceed the shear wave speed. Therefore Rayleigh wave speed is usually located in the range of 85-95% of the S-wave velocity.

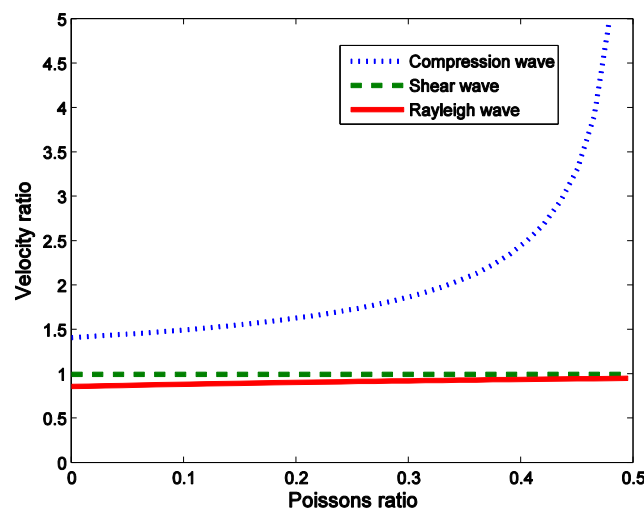


Figure 4 - The effect of Poisson's ration on seismic wave speeds

**Young's modulus** – A measure of the stiffness of a material. It is calculated using the tangent modulus of the initial, linear portion of the stress-strain curve. As stiffness (of both track and subgrade) is the main criteria used for quality control during construction, Young's modulus is a highly influential parameter in the generation and propagation of railway vibration.

At large strains soils behave non-linearly because shear modulus depends highly on strain. Although large strains may occur in geotechnical engineering applications such as

pile driving, blasting or on off-shore oil rigs, in the case of ground vibration from railways, soil particle deformation is typically very small in comparison to its dimensions. The magnitude of strain experienced by the soil during train passage is therefore low ( $10^{-5}$  %) and can be modelled using ‘small strain’ theory. This allows for the soil to be considered as a linear elastic material and for the shear modulus to be considered to be equal to the ‘maximum shear modulus’.

**Damping** – A measure of the rate at which energy is reduced as it disperses and passes through a material. The total damping ratio is composed of geometrical and material damping and has a non-linear relationship with frequency. This frequency dependence makes damping modelling more complex for time domain modelling in comparison to frequency domain modelling. Regarding in-situ soils, material damping is typically greatest in the upper layers and reduces with depth. This is because the soil particles in the upper layers are less compacted, meaning the wave loses greater energy as it passes through the air voids. Furthermore, if a soil is saturated then it may exhibit elevated viscous damping at high frequencies. Regarding the track, damping is caused by the ballast and, if present, by a combination of rail pads, under-sleeper pads and ballast mats.

### **The role of the train**

Vibration generated at the wheel/rail interface propagates both into the track/soil, and also vertically upwards into the train vehicle. It is important that vehicle vibration is minimised because it affects passenger comfort, which is often a primary track design requirement (especially on high speed lines). Therefore trains are typically fitted with two suspension systems that are connected via a bogie. The suspension system connecting the wheel and bogie is the first suspension system and the second suspension system connects the bogie and car body ([22]). Originally this design was used predominantly to improve passenger comfort on high speed trains, however is now becoming common among other train types such as intercity locomotives.

Vibration Frequency	Effect on Passenger	Ref
0.8-8Hz	Reading & writing impairment	[24], [23]
1.25-6.3Hz	Reading impairment	[25]
0.25-0.32Hz	Motion sickness	[24]
	Other	[23], [24], [25]

Table 3 – Vehicle vibration effects on passengers

Passenger comfort is often prioritised on high speed lines because the high speed market is positioned towards business and luxury customers. In comparison, for freight lines, passengers are typically not carried, so comfort is not prioritised. Therefore, particularly for business travellers, it is important that the train provides an environment conducive to work (i.e. reading and writing capabilities are not impaired) – see Table 3.

Low frequency lateral vibration in the range (0.8-8Hz) was found to interfere with reading and writing tasks by [26], with frequencies below 5 Hz proving particularly influential. Similar findings were made by [27] who found that passenger reading speed was impaired between 1.25 and 6.3 Hz, with maximum interference at 4Hz. Further studies [24] were conducted and it was found that passenger writing ability was also effected by low



frequency vibration, however the duration of vibration was also important. It was also found that the probability of spilling liquid from a hand-held cup was also affected by frequencies in the same range.

In addition to facilitating reading and writing tasks, it is important that railway travel does not interfere with sedentary tasks [28] or cause passengers to experience motion sickness. Therefore [29] analysed 4,000 commercial train passages and found that tilting trains induced higher levels of motion sickness than non-tilting trains. Additionally, it was found that vibrations in the 0.25-0.32Hz range were most influential in causing motion sickness.

A challenge with assessing passenger comfort is that results are highly dependent on the individual under investigation. Therefore the majority of studies are performed under physical test conditions and used to derive empirical exposure relationships. These experiments are often undertaken in lab conditions where the subjects experience vibrations at predetermined and discrete frequencies. Despite this, train passengers are routinely subject to a spectrum of vibration frequencies that are constantly changing. Therefore, with the end goal of reducing passenger discomfort, the vibration levels entering the train vehicle must first be modelled and then abatement measures investigated.

To model vehicle vibration, a time domain multibody modelling approach can be used. For example, [30] developed a numerical model to simulate the vibrations of a high speed train passing over a bridge. The model results were assessed using maximum acceleration criteria and a comfort index, and it was found that rail roughness had a significant effect on passenger comfort. Alternative evaluation approaches are outlined in [31], [32],[33], [34], [35], [36], [37].

### **Mitigation of vehicle vibration**

Solutions to the reduction of vehicle vibration [38] can be divided into design solutions and maintenance solutions (Table 4).

<b>Mitigation of vehicle vibration</b>		<b>Refs</b>
Design solution	Reduction in unsprung mass	[38]
	Better wheel/axle design, braking control or higher performance materials	[39]
	Resilient wheels	[40]
	Active vibration control systems	[41]
Maintenance solution	Assess wheel shape	

Table 4 - Strategies for the mitigation of vehicle vibration

### **The role of the track**

The role of the track is to support the moving vehicles and distribute their forces into the supporting soil. A key factor is track stiffness. If it is too low then the soil and ballast experience increased deformation and the locomotives require additional power to traverse the track. Alternatively, if the track stiffness is too high then the wheel stresses are concentrated within a small area on the rail thus generating corrugation [39]. Additionally, if stiffness varies radically over track sections (e.g. transitions) then this also causes elevated track deterioration and vibrations [40].

Idealised track stiffness values are different between countries, however these are typically based on expected train types (e.g. freight or high speed). Therefore, for lines that facilitate different types of rolling stock, it is difficult to determine an ideal track stiffness. Thus, track vibration and degradation can be problematic in such cases. Two influential track imperfections (not solely caused due to track stiffness) that contribute to track vibrations are:

**Wheel out-of-roundness** – Wheels become out-of-round as a result of the manufacturing process, or from repeated loading at high frequency contents [41]. The most common manifestation is the formation of wheel flats, caused primarily by train braking/deceleration. As a result, a high frequency impact force is generated at every rotation when the corners of the wheel flat impact the rail. Recently, this has been highlighted as one of the key factors contributing to track vibration [42], [43].

**Rail irregularities** – Changes in lateral/vertical geometry, or stiffness associated with the rail. These may occur due to many factors, such as switches, welds, track settlement, ballast deterioration, ballast-slab transition zones and excessive wear [6].

As mentioned earlier, there are numerous dynamic excitation mechanisms that contribute to railway vibrations [45], many of which are outlined in Figure 5. It can be seen that the excitations associated with the locomotive are found at low frequencies and the excitations associated with the wheel, rail and track are found at higher frequencies. These higher frequencies often manifest themselves in the form of air-borne noise, while the lower frequencies generate ground vibration. Bands have been drawn on Figure 5 to illustrate a variety of perception threshold levels, in accordance with BS ISO 14837 [46]. It should also be noted that train-track resonant conditions can develop where the natural track frequencies are excited. These effects and their interaction with the underlying soil are still not fully understood, and are currently the focus of much research [47].

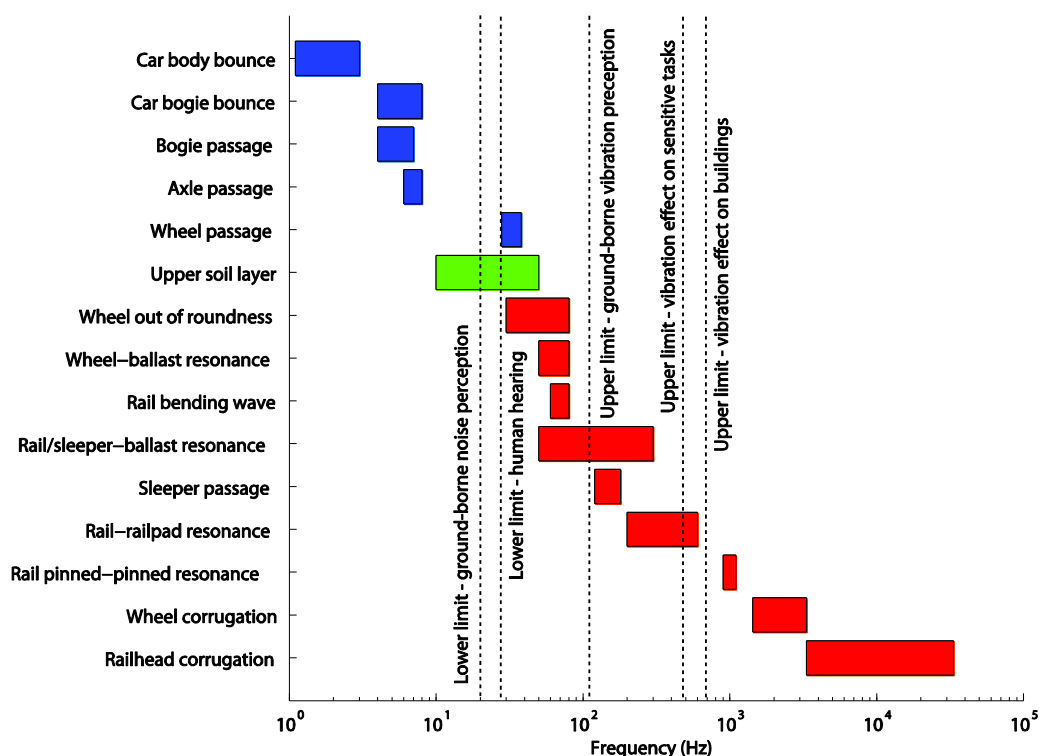


Figure 5 – Typical frequency ranges of excitation

### **Analytical track modelling**

Train-track interaction is the source of railway vibration and therefore it is important that it is modelled accurately. If the vibration response is modelled inaccurately, then not only the track vibration will be erroneous, potential train, ground and structural vibration calculations will also contain errors. Modelling can be undertaken using either analytical or numerical models, and can be computed in either the time or frequency domain. Furthermore, depending on project scope, a moving or stationary load can also be simulated.

Analytical models are attractive because they are more computationally efficient in comparison to numerical approaches. One reason for this is that numerical models suffer from spurious reflections at truncated model edges. For non-complex geometries such as soil masses, absorbing boundaries can be used, however, for more complex track type models, such boundary conditions are challenging to implement. Therefore, when using numerical methods, the total length of track modelled must be much larger than the region of interest.

For such reasons, analytical models have been analysed widely, particularly before the advent of modern computing technology. Early approaches to track modelling used the assumption that the rail could be modelled as a single beam (typically Euler-Bernoulli) resting on a Winkler foundation ([47], [48], [49], [50], [51], [52]). This foundation was used to describe the track or a combination of the track and soil. Therefore the track was often modelled as a homogenous material.

Single layer methods were useful for investigating the low frequency characteristics of the track [9]. Therefore further investigation was undertaken to investigate the interaction between various beams and plates with underlying soil, by [53]. It was found that model accuracy was improved by replacing the Euler beam formulation with that of a Timoshenko beam [54], [55], [56] and [57]. Timoshenko beams account for additional degrees of freedom (shear forces) in comparison to Euler beams and therefore allow for higher accuracy modelling of track dynamics.

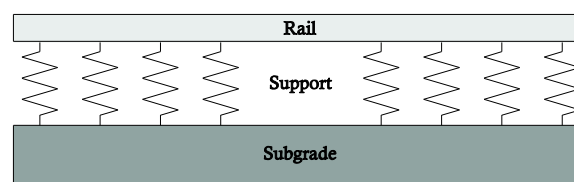


Figure 6 – Single layer track model (continuously supported)

Two layer models were also proposed and were considered an improvement over single layer models because they allowed for railpads, sleepers and ballast to be simulated [17], [58]. In the two layer model illustrated in Figure 7, the ballast and railpads are assumed to be massless, however these can be included by introducing additional layers (e.g. 3 layer models [59]). Despite this, the use of a continuous support meant that sleeper effects were not modelled. Instead the effect of the discrete sleepers was uniformly distributed over the length of the model. Although this was found to be adequate for some problems [9], and was well suited for slab track modelling, when modelling ballasted tracks it meant that sleeper passage frequencies (Figure 5) were ignored.

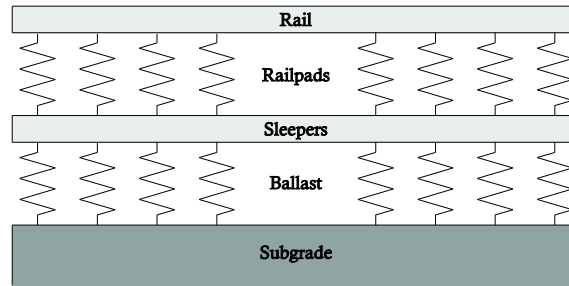


Figure 7 – Two layer track model (continuously supported)

To facilitate the inclusion of sleeper effects, discretely supported models were also developed [60], [61], [17]. Figure 8 shows a Timoshenko beam discretely supported by individual sleepers with viscous damping.

It should also be noted that the track support conditions (subgrade) have a significant effect on track stiffness and therefore have been found to affect track response. For the models previously mentioned, the track base has been assumed rigid. Although this may be suitable for tracks where the supporting material has a stiffness comparable to the track ([62]), discrepancies occur when the supporting material is soft. Therefore research has also been undertaken into modelling the subgrade as an elastic half-space [63], [64], [65], [66], [67].

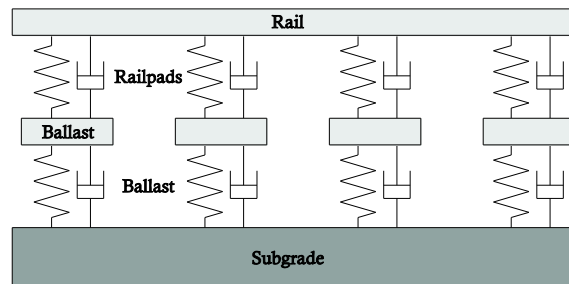


Figure 8 – Two layer track model (discretely supported)

A common method for analysing the frequency characteristics of railway tracks is to calculate the track receptance (track deflection per unit of stationary input force). This transfer function describes the propagation of energy through a system for a set range of frequencies. It is used because it provides a useful tool for investigating the natural frequencies of the track. When known, these frequencies can be dampened accordingly.

The receptance results for the various types of frequency domain track models discussed previously are shown in Figure 9. On the figure are drawn the typical upper thresholds for human (5-80Hz) and structural (30-250Hz) vibration perception. It can be seen that there are discrepancies between modelling approaches for both the human and structural response ranges, thus emphasising the need to correctly model the discrete support condition and multiple layers.

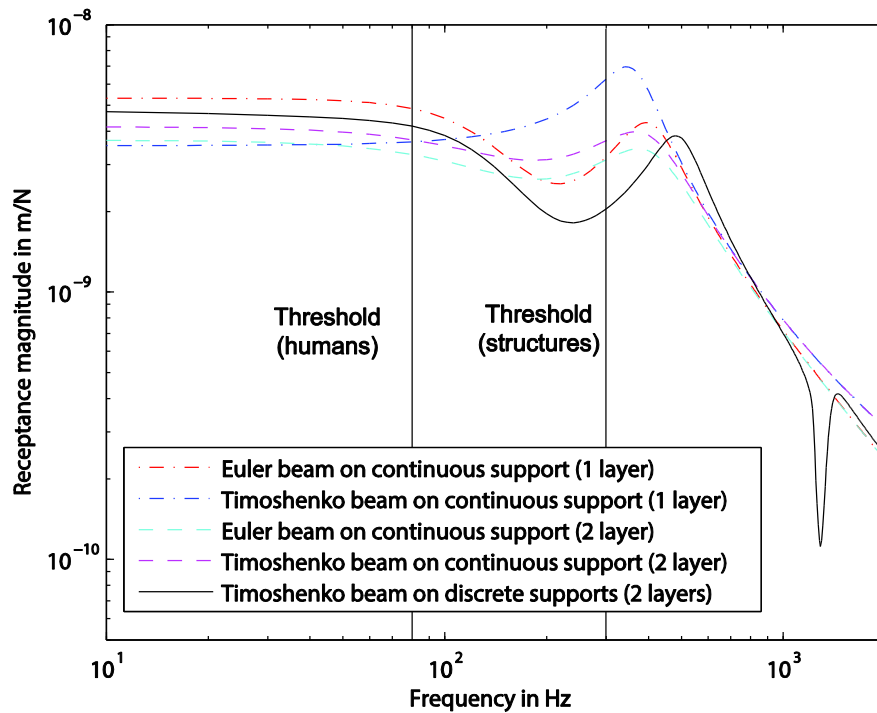


Figure 9 – Analytical track models – receptance characteristics

Although point receptances are useful for understanding general track characteristics, it is also often desirable to investigate the effect of a moving load on track response. This introduces more complexity into the analytical approach and has been studied for a moving point load, [68], [64], [47], [69], [70], [71]. In an attempt to make the excitation mechanism more realistic, the problem has also been formulated using a multi-body excitation [72], [73], [74].

### **Finite element track modelling**

Although analytical models are well suited to predicting the response of the straightforward 2D track geometries mentioned previously, they are less well suited for including multi-body vehicle effects and track/wheel irregularities. To overcome these limitations the track has frequently been modelled using more versatile numerical methods. In particular, the finite element (FE) method has proven a highly applicable tool.

A wide range of proposed FE track models were reviewed by [9]. Similarly to the analytical approaches (Figure 6 - Figure 8), these can be divided into two dimensional, one layer, two layer [75], [76], and three or more layer models [77], [78], [79], [46], [80], [81]. Multi-body vehicle modelling approaches were then used to more realistically replicate the frequency content of excitation from a moving train ([82], [83], [21]). It was found that upgrading the approach from a basic moving point load improved model accuracy drastically [82], and therefore a fully integrated train-track system could be simulated (e.g. Figure 10). Similarly, this allowed for the investigation of vibration response due to wheel flats [84], rail irregularities [75], [85], [86], ballast-slab transition zones [87] and other defects [88].

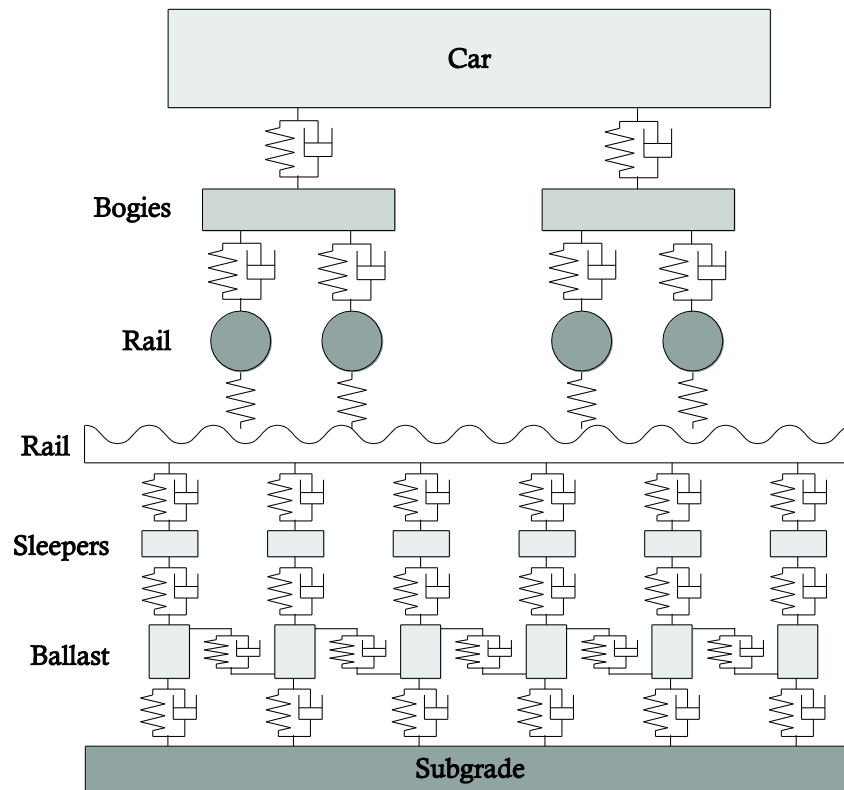


Figure 10 – Coupled train-track model

In a similar manner to analytical model development, research has been undertaken in an attempt to include subgrade stiffness (i.e. soil) contributions in track response calculations. The difference between these types of track models and the ones discussed in the ground modelling section later in this report, are that the track response is of interest, not the far field ground vibrations. Therefore the soil model is included solely to increase the accuracy of the track model. The most common approach to achieving this is to model the ground as a linear elastic half-space. This approach was undertaken by [89] to study critical velocity effects. A challenge of this approach is that computational time is increased and absorbing boundary conditions are needed to terminate the unbounded soil domain.

Another notable assumption associated with the previously mentioned models is that the track response can be reduced down to two dimensions. A challenge with this approach is that the stress distribution from the rails is then uniformly distributed over the sleeper width. This is unrepresentative of the physical problem. To overcome this, [90], [91] and [92] modelled the track components in three dimensions. An advantage of this was that none of the track material properties required tweaking to account for the 2D stress distribution approximation. To overcome some of the aforementioned challenges, it should be noted that a variety of commercial train-track modelling software is also available (e.g. Vampire, Simpack and Adams/rail).

Model type	Significance	Ref
Single beam (typically Euler-Bernoulli) on Winkler Foundation	Analytical – 2D	[48-53]

Single beam (Timoshenko)	Analytical – 2D	[55-58]
Two layer models	Analytical – 2D	[17], [59]
Three layer models		[60]
Discretely supported models	Analytical – 2D	[61], [62], [17]
Elastic half-space models	Analytical – 2D	[64-68]
Moving point load	Analytical – 2D	[69], [65], [48], [70-72]
Multi-body excitation	Analytical – 2D	[73-75]
Two layer models	FEM – 2D	[76-77]
Three layer models	FEM – 2D	[78-80, 47, 81-82]
Three layer models	FEM – 3D	[94, 93]

Table 5 – Summary of track modelling approaches

### **Ballast modelling**

[9] commented that if track vibration is to be better understood, ballast behaviour must first be further investigated. This is particularly important for investigating and improving ballast maintenance. Therefore, in recent years, there has been a rapid increase in ballast modelling, with the discrete element method (DEM - [95]), proving a highly suitable approach [96-99]. This is because ballast particles are typically large (approximately 40mm) and thus cannot be realistically modelled as a continuum material. Therefore the DE approach allows for the simulation of a large body of individual ballast particles, which can be used to investigate its abrasive, compactive and crushable nature. A challenge with DE modelling is that the complex geometries and non-linear nature of individual ballast particles leads to high computational demand. Therefore, current research is focused on reducing this demand, with a goal of integrating DE ballast models within a FE track modelling framework [100-101].

### **Mitigation of track vibration**

Track vibrations are undesirable from both passenger comfort and environmental standpoints. Additionally, as the wheel/rail interface is the source of all railway vibration, it is often efficient to isolate the vibration as close to the excitation location as possible. This is typically done using either permanent or temporary measures.

Temporary measures are approaches such as rail grinding to reduce the imperfections at the interface, thus creating a smoother contact between wheel and rail. Alternative permanent measures include the use of continuously welded rail technology which eliminates the discontinuity associated with rail joints. This approach is becoming widespread, particularly on high speed lines. Another permanent approach is to insert highly damped pads between track components [102]. For example rail pads can be inserted between rail and sleepers to reduce the vibration and stresses between components. Sleeper pads act in a similar manner, but are placed between sleepers and ballast. Both rail and sleeper pads act to dampen high frequency vibration and shift the spectrum to a lower range. This helps to reduce the overall track vibration levels, however these low frequency vibrations are more likely to resonate with structure/buildings close to the track. It should also be noted that slab tracks can be used instead of ballasted tracks,

and facilitate a reduction in vibration levels due to their increased stiffness. Additionally, floating slab tracks are often used in tunnels to provide additional isolation.

### **The role of the ground**

The ground acts as the transmission path for vibration that can eventually reach nearby structures. Thus it is important to understand and predict the characteristics of it.

A large body of work has been, and continues to be undertaken into Elastodynamic waveform modelling – see Table 6.

<b>Modelling approach</b>	<b>Comments</b>	<b>Ref</b>
Elastodynamic wave propagation	Analytical expressions for a homogenous half-space excited by a stationary point load	[103-106]
	Analytical approach to include a moving train/track model	[107-114, 48, 115, 49, 116-122]
Finite difference time domain method (FDTD)	Low computational effort and high performance ABC's	[123-124]
Finite element modelling	Complex geometries can be modelled	[93-94, 125, 90]
Coupled finite element – boundary element modelling	Complex geometries can be modelled using FE and large offsets modelled using BE	[126-132]
2.5D methods	Reduces computer run time	[133-134, 84, 135-136]
Pipe-in-pipe (PiP) models	Efficiently calculate underground vibration	[137-138]
Empirical approaches	For initial scoping studies	[139-141, 236]

Table 6 - Summary of Elastodynamic railway modelling research

Such problems were first analysed by [103], and later by [104-106] to develop analytical expressions to describe wave propagation within a homogenous half-space due to a stationary point load. Although these solutions proved invaluable in the validation of many complex wave propagation models proposed thereafter, the adaption of such an analytical approach to include a moving train/track model that is representative of a railway system, is challenging. The first steps towards achieving this were undertaken by [114, 48, 115, 49, 116-122] who built upon Lamb's work to facilitate moving load simulation. Further modifications were proposed by [107-113] to tailor the approach to railways. Despite this, to enable the train-track system to be modelled analytically, many assumptions must be made to reduce model complexity, particularly with respect to model geometry and excitation mechanism. Although significant research into this area is still being pursued [142], these challenges mean that it has become more common to use numerical techniques as they are better suited to simulating the aforementioned complexities.

One of the more straightforward numerical modelling tools is the finite difference time domain method (FDTD) [143,144]. It is based on approximating a strong formulation of the



seismic wave equation using a central differencing integration scheme. The domain is usually discretised into velocity and stress components that are staggered in 2D/3D space.

The advantages of the FDTD method are that it requires relatively low computational effort. This is because the velocity/stress discretisation is well structured and therefore it is straightforward to divide the workload between multiple computer processors. Also, high performance absorbing boundary conditions have been developed (e.g. perfectly matched layers) which are more straightforward to implement within the FDTD method than for other alternative numerical methods. (E.g. [145-146]). This means that the domain sizes can be reduced, therefore further decreasing the total number of model calculations.

These strengths have led to the investigation of the suitability of FDTD for railway problems to be explored. However, the FDTD method offers reduced performance in modelling domains with complex geometries and free surfaces. Therefore it is challenging to simulate railway track components, or the coupling between wheel and rail. Despite this, it was successfully used by [123] to model a railway track and embankment, with the input force obtained directly from a time history collected experimentally from a train. Alternatively, [124] used the FDTD technique to explicitly model the railway track. In this work an unpublished method to reduce dispersion within the complex track geometries was used.

To overcome the absorbing boundary challenges faced by the FDTD method, Sheng et al ([68, 147]) proposed a semi-analytical. The governing equations for the vehicle were computed in the frequency domain while the track-soil equations were solved in the frequency-wavenumber domain. This resulted in a single transfer function for the overall system. It was found that this approach was more computationally efficient in comparison to alternative numerical methods [148]. It was also able to efficiently model track/wheel irregularity effects.

A weakness of the approach was that many assumptions were required to formulate the analytical expressions used to calculate the transfer functions. Therefore it was difficult to include complex changes to geometry or calculate structural vibration. This is important because detailed soil models are commonly used to appraise mitigation measures (e.g. wave barriers). Furthermore, the coupling between track and ground was based on the stress distribution between each sub-model, which should ideally be determined from in-situ experiments [149].

An alternative modelling tool to the FDTD method is the finite element method (FEM) , [150-152]. It utilises a weak form of the seismic wave equation and a key strength is that complex geometries can be modelled, particularly with the widespread availability of graphical user interfaces (e.g. ABAQUS, LSDYNA, ANSYS) associated with commercial software. This is useful for railway applications because track components and their connections can be modelled explicitly ([93-94]). For time domain modelling it allows for the straightforward investigation of track defects (e.g. rail irregularities and local defects [125]) and changes in stiffness (e.g. transitions from ballast to slab track [90]). Despite this, as with the FDTD method, for unbounded domains (e.g. soils) an absorbing boundary condition (ABC) is required to prevent reflections from edges of the domain.

For time domain FE modelling, solutions such as infinite elements [153-155] the combined FE-thin layer method (Fig. 13 – [156]), and the scaled boundary FE method [157] have been proposed. More recently, the usage of perfectly matched layers (PML) has been preferred due to its superior performance. PML is a series of layers that have identical material properties to the modelling domain, however they act to stretch both the real and

imaginary coordinate space. This serves to dampen wave amplitudes in a frequency independent and more efficient manner than previous ABC approaches [158,159,145].

A weakness of time domain simulation for 3D wave propagation modelling is the large computational expense required to compute the solution at every timestep. This results in computational run times of hours/days. Therefore, a commonly used alternative approach is to perform the analysis in the frequency domain. This requires that an eigenvalue problem is solved for each frequency of interest, resulting in reduced computational requirements. A weakness of the frequency domain FE approach is that the majority of time domain ABC's cannot be used in the frequency domain, thus making it difficult to prevent boundary effects.

To overcome these limitations the finite element has frequently been coupled with the boundary element method ([160-161] (Fig. 11). The boundary element method only requires the boundary of the domain to be meshed, and uses greens functions to model wave propagation. This means that vibrations can be modelled at large offsets which is attractive for railway applications. A disadvantage of the boundary element method is that irregular geometries within the domain cannot be modelled. This makes track modelling challenging and therefore it has been coupled with the finite element method with the aim of combining the strengths of both methods. To do so, the track region and near field are typically modelled using the finite element method, while the far field is simulated using the boundary element method. Although this approach can be formulated in the time domain [126-128], a frequency domain approach has more often been used due to reduced run times [129-132]. Furthermore, when formulating the BE equations in the frequency domain, damping is modelled in a straightforward manner using a complex valued shear modulus, while in the time domain, damping is more challenging to simulate.

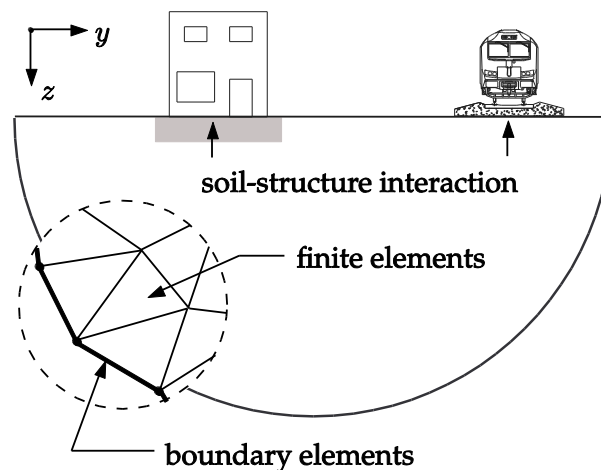


Figure 11 – The coupled BE/FE approach

A downside of the BE/FE approach is that the matrix formulation for the FE region is well structured and uniform whereas the matrices for the BE regions are sparse. Therefore it can become computationally expensive to compute the solution. Additionally, although it is novel to predict ground vibration levels at very large offsets (inside the BE region), the assessment of structural vibration is typically of greater importance in commercial projects. For areas inside the BE region, structural vibration is difficult to determine due to the difficulty of coupling a building to the surface of a BE domain [162]. Therefore, it is common for the soil and building to be modelled as two uncoupled systems [163].

One approach to reduce the computational demand of the aforementioned large 3D models is to use a 2.5D approach [133-134, 84, 135-136]. To do so the track is usually considered as invariant in the direction of train passage, thus allowing for the problem to be approximated using a 2D geometry, while accounting for 3D loading conditions (Fig 12). The solution is found in the frequency-wavenumber domain and results in much reduced computational cost in comparison to 3D models. To do so, a Fourier transformation in the direction of train passage is used:

$$\tilde{u}(x, k_y, z, \omega) = \int_{-\infty}^{+\infty} \hat{u}(x, y, z, \omega) \exp(ik_y y) dy$$

$$\hat{u}(x, y, z, \omega) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \tilde{u}(x, k_y, z, \omega) \exp(-ik_y y) dk_y$$

where  $x$ ,  $y$  and  $z$  are the coordinate axis (perpendicular to train passage, parallel to train passage and vertical respectively). To calculate the solution, a 2D model is computed for all desired frequencies ( $\omega$ ) and wavenumbers ( $k_y$ ), and then the 3D solution is recovered via the inverse Fourier transformation with respect to wavenumber. Large computational savings are made unless the number of wavenumbers is large.

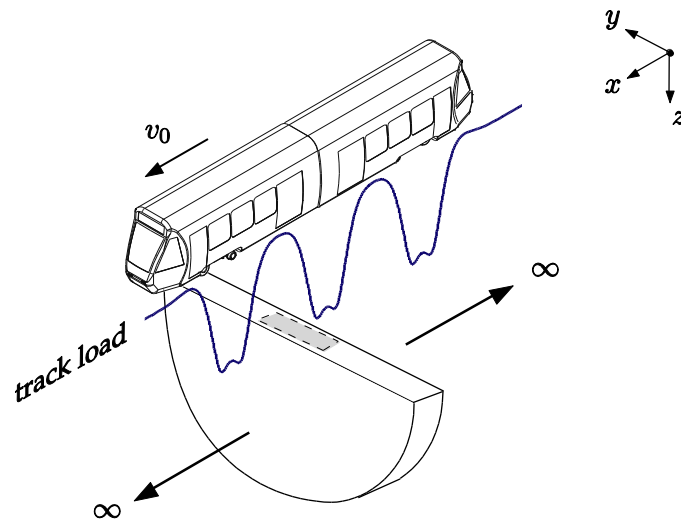


Figure 12 – The 2.5D approach

Although this approach is efficient for calculating the soil response for invariant track geometries (e.g. slab track), it is more difficult to model the stress distribution associated with discrete sleeper (i.e. ballast) tracks. One approach to overcome this limitation is to use a Floquet transform [164-165].

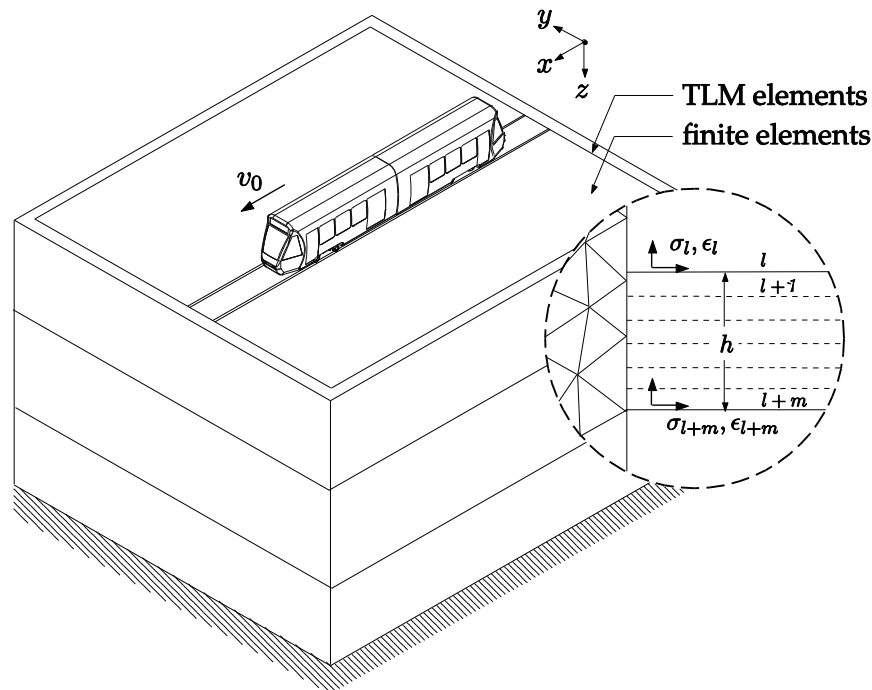


Figure 13 – The coupled FE/TLM approach

Although a broad selection of numerical approaches have been discussed to predict soil vibration propagation, it should be noted that there are many unmentioned alternative approaches. Some of these include pipe-in-pipe (PiP) models [137-138] to efficiently calculate underground vibration, empirical approaches [139-141] for initial scoping studies, and hybrid models [132], and coupled FE-meshless methods. It should also be noted that for any model to be considered valid, it must be checked against a variety of experimental results [166-168, 237].

Furthermore, one notable aspect to consider when choosing a prediction tool is the desired frequency range. If re-radiated ground-borne noise is a concern then the frequency range under consideration must be higher than for ground-borne vibration. A challenge with numerical modelling is that the maximum frequency that can be accurately resolved is related to the element discretisation. Higher frequencies require smaller element dimensions thus significantly increasing computational requirements. Therefore, sometimes analytical alternatives are attractive because they are less sensitive to changes in frequency content.

## Mitigation of ground vibration

Soil vibration can be mitigated using trenches (aka wave barriers - Figure 14a). These are excavated soil sections located in the ground at distance from the track. They are typically filled using a low density material (e.g. gas cushion [169] or polyurethane[170]) to increase the reflection coefficient between the soil and the trench, thus increasing its effectiveness. Essentially, the backfill should be of a low acoustic impedance compared to the background soil. They have been successfully used on Swedish projects in Gnarp, Stockholm, Uppsala and Saffle in the 1980's [171], and Dusseldorf, Germany [171]. Extensive numerical modelling has been undertaken into their effectiveness [18, 170, 172-176, 94], and it is consistently found that trench depth should be approximately half the length of the dominant Rayleigh wavelength.

An alternative approach is to use subgrade stiffening at strategic ground locations (Figure 14b). This serves to increase the soil stiffness and thus dampen the propagation of vibration. Possible stiffening techniques include deep vibro-compaction, deep soil mixing, stone columns, grouting or vacuum consolidation. A more comprehensive comparison between techniques can be found in [177]. Despite this, it should be noted that the deployment of such ground vibration mitigation measures is financially intensive and should be avoided if possible.

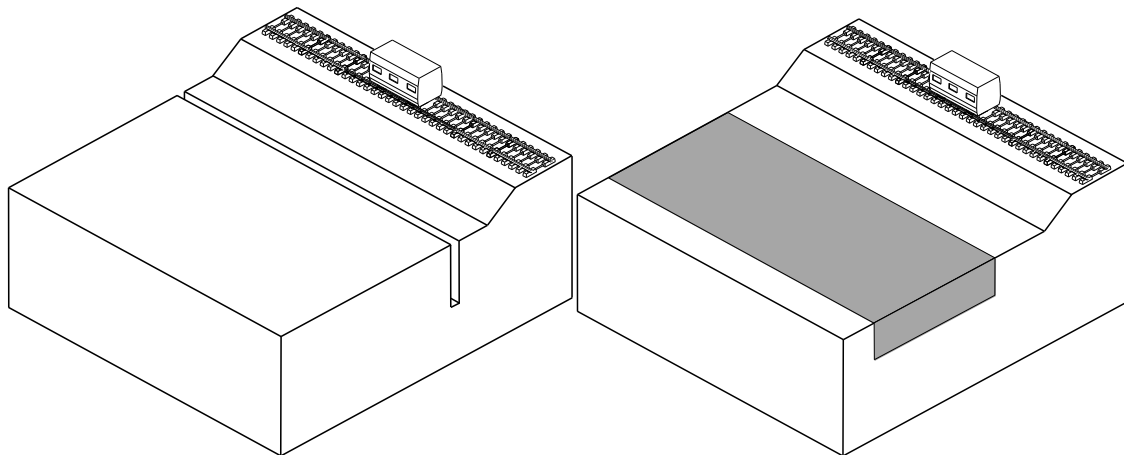


Figure 14 – Left: (a) Trench deployment, Right: (b) Subgrade stiffening

### **Critical velocity effects**

When train speeds approach the underlying Rayleigh wave speed of the supporting soil it is possible that large increases in track vibration may occur. This is an area of increasing importance for high speed rail because as operational speeds increase, the probability of train speeds approaching this ‘critical velocity’ at a particular site increases. Another important aspect is the ‘track critical velocity’, which is often described as the minimal phase velocity of bending waves propagating in the track supported by the ballast. Despite this, it is typically found that ‘track critical velocity’ is many times higher than the soil Rayleigh wave velocity.

Experimental evidence of critical velocity effects has been collected on Swedish, UK and Dutch lines and is shown in Figure 15. It is clear that as the normalised speed (train velocity/Rayleigh wave velocity) increases towards a value of 1, the track displacement grows exponentially. Trains exceeding the Rayleigh wave velocity are referred to as ‘trans-Rayleigh’ trains. These can give rise to a ‘boom’ and the creation of a Mach cone [90]. This exponential growth is a function of the initial track displacement and therefore not always problematic. For example, if track displacements are very low, then although an increase in train speed may cause a threefold increase in displacement, this value may still be below the maximum safety threshold. Despite this, it is clear that under certain, and not fully understood circumstances, that track deflections can become large (e.g. >10mm [178]).

This area is the focus of extensive research activity and is challenging due to the complex coupling between track and ground structure that contains many individual Rayleigh wave speeds and resonant frequencies, which can be analysed using a dispersion

diagram [17]. Similarly, the train excitation also generates a wide spectrum of excitation frequencies that increases problem complexity.

Another unanswered question is the behaviour of track deflections above the critical velocity. As seen in Fig. 15, data is scarce related to train speeds above this value. Therefore, although numerical models have been used to extrapolate the true behaviour of the response curve, and to approximate critical velocity values using dispersion curves [17], this is still an active area of research.

One of the challenges in modelling critical velocity behaviours is that when track displacements become large, it undergoes ‘large’ shear strain. Therefore the system no longer behaves in a linear manner and thus material behaviour must be modelled using non-linear theory [179]. This increases model complexity, makes non-linear material parameter selection challenging, and increases computational cost. Furthermore, when modelling non-linear soil effects it is usually necessary to use time domain modelling approaches rather than the frequency domain.

In practical terms many railway designers will attempt to ensure that train velocity does not exceed 0.7 times the Rayleigh wave velocity.

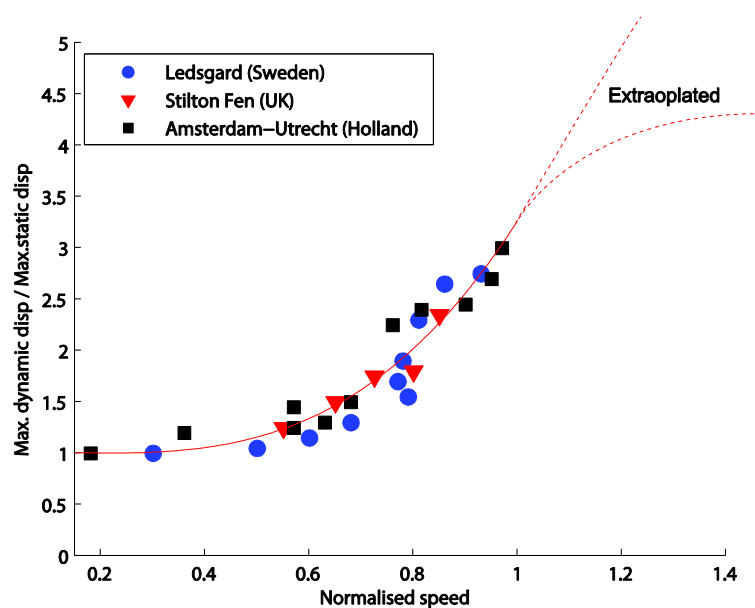


Figure 15 – Critical velocity effects (Redrawn from [178])

### Mitigation of critical velocities

A key difference between mitigating common ground vibrations and vibrations due critical velocity effects are the stakeholders involved. For common ground vibration, the levels are relatively small and the main stakeholders are the owners of buildings located close to the track. For vibration due to critical velocity effects, the key stakeholders are the railway operators and track asset managers.

Abatement measures are undertaken in the soil located beneath/within the track (active isolation) whereas for common vibration these measures are taken in the soil closer to the structure (passive isolation).

In practice, when excessive vibration levels have been encountered, the most common mitigation measures have been to reduce vehicle speed over soils with low Rayleigh wave velocities. This approach was initially undertaken at Ledsgard, Sweden where the operational line speed was reduced from 200 km/h to 130 km/h [181]. This meant that the normalised speed with respect to the Rayleigh wave speed was reduced, and therefore track deflections were reduced. A disadvantage of this approach is that it is expensive over long time scales because line capacity is reduced. Therefore, in the long-term, an engineering solution is preferable.

An engineering solution is to increase soil stiffness. Historically, one of the most commonly recorded soil properties is undrained shear strength. Therefore, to determine the necessary stiffness required to safeguard against critical velocity effects, the following equation can be used:

$$E = K_c \cdot c_u$$

Where the correlation factor  $K_c$  is dependent on plasticity index. Table 7 shows typical values for an overconsolidation ratio of between 1 and 2. In practise, it is often assumed that for design purposes, the “design critical velocity” is limited to 0.7 times Rayleigh wave velocity. Figure 16 shows design critical velocities for various undrained shear strength and plasticity index values (density=1800 kg/m<sup>3</sup>). It is observed that:

1. Higher plasticity soils can be more problematical than low plasticity soils
2. Further consideration needs to be given to soft/weak soils. For example, [182, 115] reported unexpectedly poor performance of weak soils in Sweden. This could be due a build-up in positive pore water pressure in the saturated clay – giving rise to a reduction in the effective stress and consequently a short term reduction in the encountered undrained shear strength. The latter would then reduce the Rayleigh velocity and thus the “design critical velocity”. This latter mechanism, well known in highway construction design, has not been discussed in the railway environment.

Plasticity Index, $I_p$ (%)	Correlation factor, $K_c$
< 30	600 – 1,500
30 – 50	300-600
> 50	175

Table 7 – Soil correlation factors based on plasticity index (taken from [183])

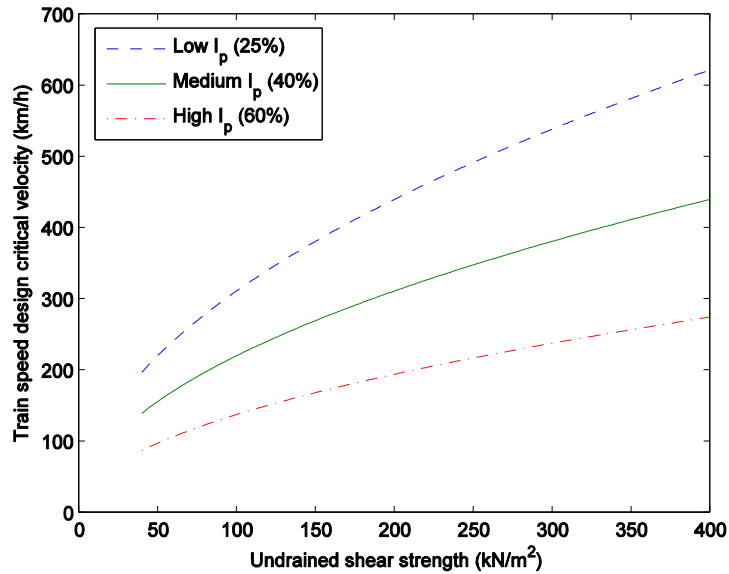


Figure 16 – Design critical velocity (saturated soil)

Note that ground improvement techniques used for critical velocity mitigation are similar to the subgrade stiffening described for common vibration abatement, but placed beneath the track, rather than at soil locations outwith the track. The purpose of this is to increase the underlying Rayleigh wave speed. At Ledsgard, lime/cement columns were placed to depths of between 7m and 13m below the track. This solution was found to significantly reduce the track deflections. Alternative solutions include stone columns, piles and the application of polyurethane [184, 185].

## **The role of structures**

Vibrations generated within the train/track propagate through the soil before reaching man-made structures. They then cause structural vibration and in-door noise, which is a growing problem internationally [186]. The soil serves to dampen vibrations, particularly at high frequencies so the majority of the vibration spectrum arriving at these structures is below 100 Hz (Figure 5). As the natural frequencies of most buildings are located below 10Hz, the soil shifts the track vibration to a range more likely to resonate with structures. Despite this, for intercity trains and high speed trains passing on traditional at-grade, cutting or embankment tracks, noise is usually more dominant than vibration. This is partly because on these lines, due to compulsory land purchases during construction, few buildings are located within 60m of the track. Locations out-with this distance are still subject to railway noise, however vibrations are sufficiently dampened thus preventing disturbance.

In comparison, there are two cases where railway vibration is an increasing concern: (1) underground high speed lines, and (2) urban tramways. In these cases it is more common that vibrations will arrive at nearby structures with the amplitude and frequency characteristics to cause negative structural effects. Therefore, vibration



propagation/prediction for underground high speed lines, and urban tramways is the focus of this section.

Underground high speed lines are problematic because noise is confined within the tunnel, however vibration propagates to the surrounding soil and interacts with structures located above the tunnel. Although many railway tunnels are located deep below the earth's surface, buildings with deep foundations may experience elevated vibration levels. For these cases, S-waves dominate the response at distances close to the tunnel, however as distance increases, P-wave can become destructive. In comparison, for at-grade tracks, Rayleigh surface waves carry the majority of vibration energy.

The second case is that of urban tramways. Although trams move with low speeds, they can have high unsprung masses [187], and their urban setting means that buildings are located very close to the track. Therefore, there are a large number of stakeholders potentially affected.

The third case is that of freight traffic which moves with low speeds, and typically has very high unsprung mass [188,189]. This generates high amplitude, low frequency vibration (+20 dB below 20 Hz) that can propagate to large distances from the track. It is difficult to mitigate such vibration because the long Rayleigh wavelengths propagate deep within the soil.

Irrespective of the proposed train-track system, when a new line is planned it is usually implicit that a vibration assessment is undertaken. This involves a scoping assessment, and depending on its findings, is followed by a more detailed assessment at any sensitive sites.

### **Scoping/empirical assessment**

Scoping assessments are used during (or sometimes before) the planning stage of a new line and are most frequently based upon empirical relationships and previous experience. It is typical for such empirical relationships to be the only form of vibration study undertaken on a new line as numerical modelling remains primarily confined to research.

A commonly used approach is that outlined by [139], [190] where a recipe is used to adjust a base vibration-distance curve to calculate absolute vibration levels within structures close to the line. The recipe is based on vibration levels recorded from previous train passages and provides a straightforward method for assessment. An important issue raised is that existing buildings can amplify vibrations significantly (i.e 100% increase), depending on their type. This is a significant simplification as many structural and furnishing factors may need to be taken into account for a specific structure. An alternative assessment and calculation procedure is outlined by [191] which was derived from TGV passages recorded in France and used to calculate tunnel vibrations for a UK rail project.

Similarly, [192] statistically analysed vibration levels on Swedish and Norwegian rail lines to develop a methodology for predicting high speed rail vibrations in Oslo. Also, [193] outlined an empirical model based on experimental results obtained in Sweden. The model was able to predict one second r.m.s (root mean square) values based on receiver distance, wheel force, vibration attenuation and train speed. Lastly, [194] proposed a scoping model that was based on straightforward analytical expressions and then calibrated using experimental field data.

### **Detailed prediction**

Detailed prediction models may involve a variety of experimental and numerical methodologies. They are deployed for highly sensitive sites where either high accuracy is required, or the problem is bespoke.

For tramway vibration, indoor noise (i.e. reradiated groundborne noise) is of primary concern and when undertaking prediction it is common to use a physical experiment approach [139], 195-197]. This involves performing impact tests at the location of the potential tram line and measuring the vibration frequency response in the floors of surrounding buildings (typically vertical vibration at floor mid-spans). If the impact force is also recorded then a transfer function can be calculated. This transfer function is combined with a transfer function simulating the vibration response of the newly proposed train/track system under investigation (calculated numerically, experimentally or empirically [198]. The final transfer function represents the frequency response of the buildings to the new line, and can be used to estimate indoor noise levels.

The strength of using an experimental approach is that the exact soil wave propagation characteristics of the test site, and the coupling between soil and structure are included in the calculation. If an alternative numerical approach was used the soil properties at the test site would need to be acquired and then used to recreate a model of the test site and proposed new track [199]. As existing soil records are usually not sufficiently detailed to determine the numerical parameters required for detailed investigation, in-situ geophysical tests must be performed (although, some simple relationships between conventional SI data and numerical input data were given above). The most common acquisition method for this application is multi-channel analysis of surface waves testing (MASW), which enforces the assumption of horizontal homogenous soil layering, and challenging in the presence of high gradient topographies (i.e. building foundations). It has also been well documented that the optimisation approach performed during post-processing results in a non-unique solution [200]. Furthermore, measurement procedures related to the calculation of low frequency damping are not yet well developed.

In addition to the errors introduced during MASW testing and post-processing, there are also potential errors introduced by the numerical soil modelling approach (e.g. boundary effects, non-linear effects...etc) [201]. Therefore, in comparison to the experimental transfer function approach, numerical methods introduce unnecessary risk into the prediction process. Similar challenges arise relating to the modelling of the soil-structure interaction/coupling [202-204], which if modelled explicitly using numerical methods, requires extensive computational effort. Furthermore, it should also be noted that the time required to perform a numerical prediction is often longer than an experimental one. This is because the deployment of numerical models is computationally intense and requires more time than the experimental transfer function investigation. This effect can be magnified if high accuracy predictions are required (e.g. for a hospital, manufacturing plant or concern hall), because for this single site, the same model may need to be computed numerous times to assess the effect of input parameter uncertainty on vibration levels [205-206].

For underground high speed lines, it is possible to use a similar experimental approach, where a borehole is drilled to the depth of the proposed tunnel, and an impact performed at the base. This method is impractical if the water table location is likely to change after construction (e.g. tidal effects), or if large offset measurements are required.

Similarly, if the proposed site cannot be accessed then it may become necessary to use numerical modelling.

The numerical modelling approaches used to predicting structural vibration from underground lines [137-138], 207-208, 124, 129] are similar to those discussed in the previous soil propagation section. This is because it is common to predict soil vibration using numerical methods and then use an empirical transfer function based on the building type [139, 209-210] to calculate the structural vibration and thus indoor noise. This approach is taken because the computational demand of coupling a building to a train, track and soil model is prohibitive, with many site-specific (and thus challenging to quantify) parameters.

Numerical models remain attractive because, assuming a suitable model exists, the assessment process may be cheaper and quicker to deploy, particularly if the consultant is not based near the projects' geographical location. Despite this, to achieve high accuracy, numerical methods often require geophysical investigations to determine input parameters (e.g. soil, track and building characteristics). If these parameters can be obtained, then rather than potentially compromising the extensive effort required to simulate soil vibration by combining it with an empirical approach for building vibration, there is a desire to model the entire physical problem numerically. Thus, with the aim of superseding the current approach, much research is being undertaken to numerically predict the entire wave propagation path from train wheel to structure and its ultimate conversion into indoor noise. If the entire system can be correctly modelled then it becomes possible to better investigate mitigation measures and the effect of track changes directly on indoor noise. Despite this, if/when numerical modelling overcomes these problems; another challenge is the quantification and acquisition of additional structure-soil coupling parameters.

In an attempt to model the entire wave propagation path, [211] developed a finite element model of a building and used a harmonic load to simulate the vibrations generated by an underground train. Alternatively, [212] used a sub-modelling approach to couple a 2D building with a 3D underground train, track and soil model. This frequency domain approach allowed for the rapid calculation of the building response and included rail irregularities.

[213-214] also predicted structural vibration and in-door noise, but using a FE/BE approach. The track was modelled using the finite element method and the boundary element method was used for the soil response. To simulate the structural response a sub-structuring approach was used and the in-door acoustic response calculated using spectral finite element methods.

As vibration prediction models become more complex, achieving higher accuracy, software computational times increase. The four cornerstones of a prediction model are interlinked and are shown in Figure 17. As a counterexample, for a simple empirical model, the execution time is negligible, the usability is high meaning extensive training is not required, and parameter requirements are low meaning few input parameters require investigation prior to execution. However, these advantages are offset by the low model accuracy.

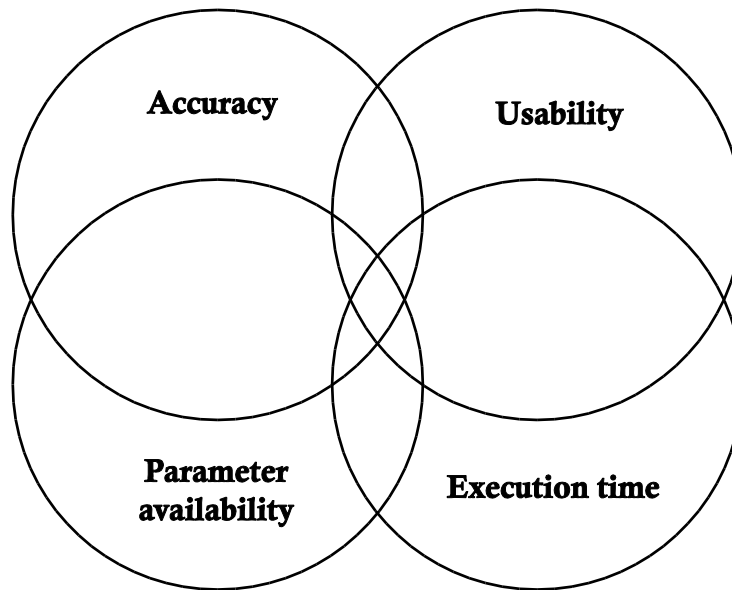


Figure 17 – The four desirable characteristics of a vibration prediction model

### The effects of structural vibration

If the natural frequencies of the ground vibration and a nearby structure are similar, then the structure will vibrate. This can impact on humans dwelling within this structure [215] through three primary methods:

1. Whole body response
2. Low frequency noise from vibrating floors and walls
3. Noise/vibration from other objects, e.g. rattling of windows, doors and furniture

A large body of research has attempted to derive exposure–response relationships to determine the level of annoyance of building residents to railway vibrations [216,217]. For example, [218,188] derived relationships for freight trains, [219] analysed annoyance for passenger trains, and [220] analysed the effect of the quantity of train passages on annoyance.

Additionally, vibration is perceived differently depending on the subject’s reference value of vibration. For example, for an existing railway line that is upgraded and vibration levels are less than 25% greater than the levels previously experienced, residents are unlikely to perceive the increase [184].

There are many national and international standards which provide guidelines for acceptable vibration levels [221,188]. Vibration levels are typically calculated differently depending on country, however in general, can be divided into three broad types: Maximum running RMS (root mean squared) values, energy equivalent RMS descriptors and VDV (vibration dose values) as used in the UK [222]. These types are also subject to individual frequency weightings and directional weightings depending on country. Some of the most commonly used international metrics to predict vibration levels include: KB [223], VDV [224] and VdB [139]. Furthermore, investigations into the relationship between noise and vibration and have found that:

1. When combined, the presence of both noise and vibration can affect the human perception of each individual component.
2. Vibration has little effect on perceived noise levels [225], but that noise has an effect on perceived vibration levels [226]. This has been found in both the lab and in field testing [227].
3. No standards or guidelines currently include combined noise-vibration relationships.
4. Human perception to vibration (and noise) is highly subjective and dependent on many factors [228-229, 221].

### **Mitigation of structural vibration**

If a structure is planned to be constructed near a railway line then rather than implement a ground abatement measure (e.g. trench), it is also possible to isolate the vibration via structural design. This approach is most commonly employed for buildings above underground railway lines [230].

Many structural abatement type approaches stem from earthquake engineering principles. The main principle is to isolate the building from the ground vibration, typically, using a spring type mechanism (aka base isolation) to alter the frequency response of the system. Despite this, isolation is complex because the soil-building response is dynamic and effected by the foundation properties, isolation method, structural flexibility and building damping [13]. Approaches to isolation include [13]:

1. Mounting the new building on springs [230–234]
2. Increasing the thickness of the lower floor [235].

### **Conclusions**

From a synthesis of over 200 scientific papers, a number of new conclusions regarding ground borne vibrations and Rayleigh waves from classic and high speed railways have emerged:

(1) Vehicle vibration is an area that affects passenger comfort. The implications of differing vibration frequency ranges have been synthesised from the literature highlighting the effect on the passengers' ability to read and write – typically it is worst at around 4 Hz. Motion sickness occurs at even lower frequencies in the range 0.25-0.32 Hz. Mitigation strategies have then been synthesised from the literature – e.g. reduction in unsprung mass and active control; systems.

(2) It is well established that track vibrations are undesirable because they cause riding quality and safety concerns; and increase track degradation. A large volume of papers have been written on predicting track and ground borne vibrations using techniques ranging from classical mathematical analyses to finite element (FEM) and boundary element methods (BEM). The many analysis techniques have been summarised in tabular format drawing attention to 2-D, 2.5-D and 3-D analyses. An attempt has been made to present a clear and simplified set of guidelines for designers. An obvious conclusion is that the more complex 3-D models are computationally hungry.

(3) Literature refers to the 'critical velocity' of both the train and the track. Although difficult to quantify in absolute terms, it has been demonstrated that if the train exceeds

both the Rayleigh wave velocity and the ‘track critical velocity’ ( $>$  Rayleigh velocity) then the train will be subject to large amplitude vibrations and a Mach cone wave propagation pattern will develop.

(4) It is now clear that for design and operational purposes, that trains should be operated below the Rayleigh wave velocity.

(5) The review of papers demonstrates that conventional site investigations (SI) do not provide the relevant data for estimation of Rayleigh velocities. Despite this, for the first time, using work by Jamiolkowski et al. [183], a relationship was developed relating a conventional SI to input parameters for the calculation of Rayleigh wave velocity, linked to soil plasticity. From this analysis, for the first time, estimates of ‘critical velocity’ have been made, albeit these must be seen as embryonic at this stage in the state-of-the-art. The ‘critical velocity’ was calculated at 0.7 of the Rayleigh wave velocity.

(6) At the proposed ‘critical velocity’, calculated as 0.7 times the Rayleigh wave velocity, exaggerated problems due to the ‘track critical velocity’ should therefore be avoided.

(7) The ground provides the propagation path between track and nearby buildings. Structural vibration is an increasing concern in urban environments. It has been shown that structural vibrations become more noticeable when combined with noise pollution. The FRA proposes that structural building resonance can amplify ground borne vibrations by over 100%.

(8) Various vibration mitigation strategies have been identified ranging from ground improvement to wave isolation measures such as trenches backfilled with low acoustic impedance backfill.

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